RESEARCH ARTICLE OPEN ACCESS

Five Dimensional Bianchi Type-V Space-Time in *f (R,T)* Theory of Gravityw

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Abstract
We study the spatially homogeneous anisotropic Bianchi type-V universe in *f(R,T)* theory of gravity, where R is the Ricci scalar and T is the trace of the energy-momentum tensor. We assume the variation law of mean Hubble parameter and constant deceleration parameter to find two different five dimensional exact solutions of the modified field equations. The first solution yields a singular model for $n \neq 0$ while the second gives a non-singular model for $n = 0$. The physical quantities are discussed for both models in future evolution of the universe.
Keywords: $f(R,T)$ theory of gravity, Bianchi type-V, five dimensional space-time, Hubble parameter.

I. Introduction

In modern cosmology, the main goal to determine the large-scale structure and study accelerated expansion of the universe. Dark energy and dark matter are two exotic components filled with most part of universe observed during astronomical observation [1-7]. The most challenging question in modern cosmology is to study the nature of dark energy is the unknown mystery in the field of relativity. To get result of such puzzle, there is requirement to modification of general relativity proposed by Einstein.

The several modification of general theory of relativity proposed recently among them few are *f(T), f(R)* and *f(R,T)* theory of gravity. The *f(T)* theory of gravity, where T is the scalar torsion has been proposed to explain current acceleration of the universe without involving dark energy (DE). M. sharif et. al. [8] considered spatial homogeneous & anisotropic Bianchi Type-I universe in *f(T)* gravity. Ratbay M. [9] has shown that the accelerating expansion of the universe understood by the *f(T)* gravity models. T. P. Sotiriou [10] had been discussed large-scale structure in *f(T)* gravity. Cosmological perturbations in *f(T)* gravity had been investigated in [11–14]. Relativistic Stars in *f(T)* gravity had been investigated in [15]. Static solutions with spherical symmetry in *f(T)* theories have been discussed in [16].

The *f(R)* theory of gravity is the another example of modified theory of gravity. Many authors have investigated $f(R)$ gravity in different contest. Carroll et. al. [17] explained the presence of late time comic acceleration of the universe in $f(R)$ gravity. Bertolami et. al. [18] have proposed a generalization of $f(R)$ modified theory of gravity by including in this theory an explicit coupling of an arbitrary function of the Ricci scalar R with the matter Lagrangian density L_m. As a result of coupling the massive particles is non-geodesic and extra force orthogonal to four velocity arises. Capozziello et. al. [19] used Noether symmetry to study spherically symmetric solutions in $f(R)$ theory of gravity. Adhav K.S. [20] discussed the Kantawski-sachs string cosmological model in $f(R)$ theory. Multamaki and Vilja [21,22] investigated static spherically symmetric vacuum solutions of the field equations and non-vacuum solutions by taking fluid respectively. Reddy D. R. K. et. al. [23] studied vacuum solutions of Bianchi type- I &V models in $f(R)$ gravity with a special form of deceleration parameter. M. Sharif and M. Farasat Shamir [24,25] exhibit vacuum as well as non-vacuum solutions of Bianchi Types-I and V Space-times in *f(R)* theory of Gravity. M. Sharif and H. Rizwana Kausar [26] discussed non-vacuum Solutions of Bianchi Type-VIo Universe in *f(R)* Gravity. M. Sharif and H. Rizwana Kausar [27] explained Anisotropic Fluid and Bianchi Type-III Model in *f(R)* Gravity. K. S. Adhav [28] obtained Bianchi Type-III string cosmological Model in *f(R)* Gravity. Along with this Plane Symmetric Solutions in *f(R)* Gravity and Plane Symmetric Vacuum Bianchi Type III Cosmology in *f(R)* Gravity discussed by M. Sharif et. al. [29,30]. Bamba et. al. [31] described the thermodynamics in *f(R)* gravity in the Palatini formalism. Lukas Hollenstein and Francisco S. N. Lobo [32] discussed exact solutions of *f(R)* gravity coupled to nonlinear electrodynamics.

New modified theory proposed by Harko et. al. [33] is the generalize version of $f(R)$ theory of gravity known as $f(R,T)$ theory of gravity, where the gravitational Lagrangian involves an arbitrary function of the scalar curvature *R* and the trace of the energy momentum tensor *T*. Reddy D. R. K. et. al. [34] studied LRS Bianchi type-II model in $f(R,T)$ theory. Pradhan et. al. [35] reconstructed $f(R,T)$ theory of gravity by taking $f(R,T) = f_1(R,T) + f_2(R,T)$. Adhav [36] studied exact solutions of locally rotationally symmetric Bianchi type -I space-time. M. Farasat Shamir et. al. [37] obtained exact solutions of Bianchi Types-I &V models in $f(R,T)$ gravity with the assumption of constant declaration parameters variation law of Hubble parameters. R. L. Naidu et. al. [38] discussed FRW Viscous fluid cosmological model in $f(R,T)$ gravity. H. R. Ghate et. al. [39] studied of Bianchi Types-IX viscous string cosmological model models in $f(R,T)$ gravity with special form of declaration parameter. P.K. Sahoo et. al. [40] investigated Kaluza-Klein cosmological model in $f(R,T)$ gravity with $\Lambda(t)$ by considering constant decelerating parameter they found this constant decelerating parameter which yields two different model first represent power law expansion and second shows exponential law expansion.

Modified theory of gravity creates curiosity among the researcher to study physics in higher dimension due to accelerating exposition of universe. Kaluza-Klein [41,42] give five dimensional history is unforgettable moment in the field of cosmology, which unite the gravitation field propose by Einstein in general relativity and the electromagnetic field in Maxwell theory. Cosmological and astrophysical implication of extra dimension has been discussed by Wesson and Overduin [43,44]. A numerous authors have done work on higher dimensional space-times in general relativity as well as non-symmetric unified field theory [45-50]. In recent year's super-string, super-gravity and other field theories provoked great interest in studying higher dimension space-time in theoretical physics, therefore in this direction most recent efforts have been directed to studying theories in which the dimensions of the space-time are greater than four.

II. Five dimensional field equations in $f(R,T)$ theory of gravity

The action for $f(R,T)$ in five dimensions is given by

$$S = \int \left(\frac{1}{16\pi G} f(R,T) + L_m \right) \sqrt{-g}\ d^5 x, \tag{1}$$

Where $f(R,T)$ is an arbitrary function of Ricci scalar *R* and *T* is trace energy momentum tensor of matter T_{ij}, L_m is matter lagrangian density.

The five dimensional field equations in $f(R,T)$ theory of gravity are given by

$$f_R(R,T)R_{ij} - \frac{1}{2} f(R,T) g_{ij} - (\nabla_i \nabla_j - g_{ij}\Box) f\ (R,T) = kT_{ij} - f_T(R,T)(T_{ij} + \theta_{ij})\ (i, j = 1,2,\ldots 5), \tag{2}$$

where $f_R(R,T) \equiv \frac{\partial f_R(R,T)}{\partial R}$, $f_T(R,T) \equiv \frac{\partial f_T(R,T)}{\partial T}$, $T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\partial\left(\sqrt{-g}\ L_m\right)}{\partial g^{ij}}$, $\theta_{ij} = -pg_{ij} - 2T_{ij}$

$\Box \equiv \nabla^i \nabla_i$, ∇_i is the covariant derivative.

The energy momentum tensor for perfect fluid yields

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \tag{3}$$

where $u_i = \sqrt{g_{00}}(1,0,0,0,0)$ is five velocity in co-moving coordinates and ρ, p are energy density and pressure of the fluid respectively.

Contracting the above field equations (2), we have

$$f_R(R,T)R + 4\Box\ f_R(R,T) - \frac{5}{2} f(R,T) = kT - f_T(R,T)(T + \theta), \tag{4}$$

Also field equations (2), take the form

$$f_R(R,T) R_{ij} - \frac{1}{2} f(R,T) g_{ij} - (\nabla_i \nabla_j - g_{ij}\Box) f_R(R,T) = kT_{ij} + f_T(R,T)(T_{ij} + pg_{ij}), \tag{5}$$

Harko et. al. [33] gives three class of models out of which we used $f(R,T)=R+2f(T)$ for this models equation (5) can be written as

$$R_{ij}-\frac{1}{2}Rg_{ij}=kT_{ij}+2f'(T)T_{ij}+[f(T)+2pf'(T)]g_{ij}, \tag{6}$$

where overhead prime denotes derivative w.r.to. *T.*

We also choose (*G = c =1*) and

$$f(T)=\lambda T\,, \tag{7}$$

where λ is constant.

III. Exact solutions of Bianchi type-V space-time in V_5

In this section we find exact solutions of five dimensional Bianchi-V space-time in $f(R,T)$ gravity. The line element of Bianchi type-V space-time in V_5 is given by

$$ds^2=dt^2-A^2(t)dx^2-e^{2\alpha x}[B^2(t)(dy^2)+C^2(t)(dz^2+du^2)] \tag{8}$$

where A, B and C are metric function of cosmic time t and α is an arbitrary constant. The corresponding Ricci scalar is

$$R=-2[\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{2\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{2\dot{B}\dot{C}}{BC}+\frac{2\dot{A}\dot{C}}{AC}+\frac{\dot{C}^2}{C^2}-\frac{6\alpha^2}{A^2}], \tag{9}$$

where overhead dot means derivative with respect to t.

From equation (6), we have obtain field equations as

$$\frac{\dot{A}\dot{B}}{AB}+2\frac{\dot{B}\dot{C}}{BC}+2\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{C}^2}{C^2}-\frac{6\alpha^2}{A^2}=(10\pi+3\lambda)\rho-2\lambda p\,, \tag{10}$$

$$\frac{\ddot{B}}{B}+2\frac{\ddot{C}}{C}+2\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}^2}{C^2}-\frac{3\alpha^2}{A^2}=\lambda\rho-(10\pi+4\lambda)p\,, \tag{11}$$

$$\frac{\ddot{A}}{A}+2\frac{\ddot{C}}{C}+2\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{C}^2}{C^2}-\frac{3\alpha^2}{A^2}=\lambda\rho-(10\pi+4\lambda)p, \tag{12}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3\alpha^2}{A^2}=\lambda\rho-(10\pi+4\lambda)p\,, \tag{13}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3\alpha^2}{A^2}=\lambda\rho-(10\pi+4\lambda)p \tag{14}$$

The metric function C is common along *z* and *u* directions hence left hand side of equation (13) and (14) are identical. The system of these four non-linear differential equations consist of five undefined functions i.e. *A, B, C,* p and ρ. Hence to find derministic solution one more condition is necessary , so we consider well known relation [51] between Hubble parameter *H* and average scale factor *a* given as

$$H=la^{-n}\,,\ \text{where}\ l>0\ \text{and}\ n\geq 0 \tag{15}$$

Subtracting equation (11) from equation (12), equation (12) from equation (13), equation (11) from equation (13), we have

$$\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{2\dot{C}}{C}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0\,, \tag{16}$$

$$\frac{\ddot{B}}{B}-\frac{\ddot{C}}{C}+\left(\frac{\dot{A}}{A}+\frac{\dot{C}}{C}\right)\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0\,, \tag{17}$$

$$\frac{\ddot{A}}{A}-\frac{\ddot{C}}{C}+\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)\left(\frac{\dot{A}}{A}-\frac{\dot{C}}{C}\right)=0\,. \tag{18}$$

On solving equations (16-18), we get

$$\frac{B}{A}=d_1\exp\left[c_1\int\frac{dt}{a^4}\right], \tag{19}$$

$$\frac{C}{B}=d_2\exp\left[c_2\int\frac{dt}{a^4}\right], \tag{20}$$

$$\frac{A}{C}=d_3\exp\left[c_3\int\frac{dt}{a^4}\right], \tag{21}$$

where c_1, c_2, c_3 and d_1, d_2, d_3 are constants of integration which satisfy the relation

$$c_1+c_2+c_3=0\,,\quad d_1d_2d_3=1\,. \tag{22}$$

Using equation (19), (20) and (21), the metric functions are

$$A=ap_1\exp\left[q_1\int\frac{dt}{a^4}\right], \tag{23}$$

$$B=ap_2\exp\left[q_2\int\frac{dt}{a^4}\right], \tag{24}$$

$$C=ap_3\exp\left[q_3\int\frac{dt}{a^4}\right], \tag{25}$$

where $p_1=(d_1^{-3}d_2^{-2})^{1/4}$, $p_2=(d_1d_2^{-2})^{1/4}$, $p_3=(d_1d_2^{2})^{1/4}$, (26)

and $q_1=-\dfrac{3c_1+2c_2}{4}$, $q_2=\dfrac{c_1-2c_2}{4}$, $q_3=\dfrac{c_1+2c_2}{4}$, (27)

Satisfying the relations

$$p_1p_2p_3^2=1\quad,\quad q_1+q_2+2q_3=0\,. \tag{28}$$

From equations (23), (24), (25),we get

$$A=a\,, \tag{29}$$

$$B=aP\exp[Q\int\frac{dt}{a^4}]\,, \tag{30}$$

$$C=aP^{\frac{-1}{2}}\exp[\frac{-Q}{2}\int\frac{dt}{a^4}]\,. \tag{31}$$

Where $p_1=1$, $p_2=p_3^{-2}=p$, (32)

and $q_1 = 0$, $q_2 = -2q_3 = Q$, (33)

IV. Some important physical quantities

The average scale factor and the volume scale factor are defined respectively as

$$a = (ABC^2)^{\frac{1}{4}}, \quad V = a^4 = ABC^2 . \tag{34}$$

The generalized mean Hubble parameter H is defined by

$$H = (\ln a)_t = \frac{\dot{a}}{a} = \frac{1}{4}[H_1 + H_2 + H_3 + H_4], \tag{35}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = H_4 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x, y, z and u axes respectively.

The mean anisotropy parameter $\overline{A}$ is given by

$$\overline{A} = \frac{1}{4}\sum_{i=1}^{4}\left(\frac{\Delta H_i}{H}\right)^2, \tag{36}$$

Where $\Delta H_i = H_i - H$

The expansion scalar θ and shear scalar σ^2 are defined as under

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{2\dot{C}}{C}, \tag{37}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij}. \tag{38}$$

Where $\sigma_{ij} = \frac{1}{2}\left[\nabla_j u_i + \nabla_i u_j\right] - \frac{1}{4}\theta g_{ij}$, (39)

From equation (15) and (35), we have

$$\dot{a} = la^{1-n}, \tag{40}$$

after integrating equation (40), we have

$$a = (nlt + k_1)^{1/n}, \quad n \neq 0 \tag{41}$$

and $$a = k_2 \exp(lt), \quad n = 0, \tag{42}$$

where k_1 and k_2 are constants of integration.

The first value of average scale factor represents the power law expansion where as the second value suggested exponential law. Thus we have two values of the average scale factors which correspond to two different models of the universe.

V. Five dimensional Model of the Universe when $n \neq 0$

In this section we study the five dimensional model of the universe for $n \neq 0$. For this singular model average scale factor a given as $a = (nlt + k_1)^{1/n}$

The metric coefficients A, B and C turn out to be

$$A = (nlt + k_1)^{1/n}, \tag{43}$$

$$B = P(nlt+k_1)^{1/n}\exp\left[\frac{Q(nlt+k_1)^{\frac{n-4}{n}}}{l(n-4)}\right],\quad n\neq 4 \tag{44}$$

$$C = P(nlt+k_1)^{1/n}\exp\left[-\frac{Q(nlt+k_1)^{\frac{n-4}{n}}}{2l(n-4)}\right],\ n\neq 4 \tag{45}$$

The mean generalized Hubble parameter and the volume scale factor become

$$H = \frac{l}{nlt+k_1},\qquad V = (nlt+k_1)^{4/n}. \tag{46}$$

The mean anisotropy parameter $\overline{A}$ turns out to be

$$\overline{A} = \frac{3Q^2}{8l^2(nlt+k_1)^{(8-2n)/n}}. \tag{47}$$

The deceleration parameter q in cosmology is the measure of the cosmic accelerated expansion of the universe and is defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = n-1, \tag{48}$$

which is a constant.
A positive sign of q, i.e., $n>1$ corresponds to the standard decelerating model whereas the negative sign of q, i.e., $0<n<1$ indicates inflation. The expansion of the universe at a constant rate corresponds to $q=0$, i.e, $n=1$. Also, recent observations of SN Ia, reveal that the present universe is accelerating and value of DP lies somewhere in the range $-1<q<0$.

The expansion θ and shear scalar σ^2 are given by

$$\theta = \frac{4l}{nlt+k_1} \text{ and } \sigma^2 = \frac{3Q^2}{4(nlt+k_1)^{8/n}}, \tag{49}$$

Thus the energy density of the universe becomes

$$\rho = \frac{1}{20(\lambda+2\pi)(\lambda+5\pi)}\left[\begin{array}{l} 2(3\lambda+10\pi)\left\{\frac{6l^2}{(nlt+k_1)^2} - \frac{3Q^2}{4(nlt+k_1)^{8/n}} - \frac{6\alpha^2}{(nlt+k_1)^{2/n}}\right\} \\ -3\lambda\left\{\frac{4l^2(1-n)}{(nlt+k_1)^2} + \frac{3Q^2}{2(nlt+k_1)^{8/n}}\right\}\end{array}\right], \tag{50}$$

The pressure of the universe becomes

$$p = \frac{-1}{20(\lambda+2\pi)(\lambda+5\pi)}\left[\begin{array}{l} (\lambda+10\pi)\left\{\frac{6l^2}{(nlt+k_1)^2} - \frac{3Q^2}{4(nlt+k_1)^{8/n}} - \frac{6\alpha^2}{(nlt+k_1)^{2/n}}\right\} \\ +\frac{3}{2}(3\lambda+10\pi)\left\{\frac{4l^2(1-n)}{(nlt+k_1)^2} + \frac{3Q^2}{2(nlt+k_1)^{8/n}}\right\}\end{array}\right], \tag{51}$$

VI. Five dimensional Model of the Universe when $n=0$

In this section we study the five dimensional model of the universe for $n=0$. For this non-singular model average scale factor *a* given as $a = k_2\exp(lt)$

Here the metric coefficients take the form

$$A = p_1 k_2 \exp(lt) \exp\left[-\frac{q_1 \exp(-4lt)}{4lk_2^4}\right], \tag{52}$$

$$B = p_2 k_2 \exp(lt) \exp\left[-\frac{q_2 \exp(-4lt)}{4lk_2^4}\right], \tag{53}$$

$$C = p_3 k_2 \exp(lt) \exp\left[-\frac{q_3 \exp(-4lt)}{4lk_2^4}\right]. \tag{54}$$

The mean generalized Hubble parameter becomes

$$H = l, \tag{55}$$

while the volume scale factor turns out to be

$$V = k_2^4 \exp(4lt), . \tag{56}$$

The mean anisotropy parameter $\overline{A}$ becomes

$$\overline{A} = \left[\frac{3Q^2}{8l^2 k_2^8}\right] \exp(-8lt) , \tag{57}$$

while the quantizes θ and σ^2 are given by

$$\theta = 4l \text{ and } \sigma^2 = \left[\frac{3Q^2}{4k_2^8}\right] \exp(-8lt) , \tag{58}$$

Thus the energy density of the universe becomes

$$\rho = \frac{1}{20(\lambda + 2\pi)(\lambda + 5\pi)} \left[\begin{array}{l} 2(3\lambda + 10\pi)\left\{6l^2 - \dfrac{3Q^2}{4k_2^8 \exp(8lt)} - \dfrac{6\alpha^2}{k_2^8 \exp(2lt)}\right\} \\ -3\lambda\left\{4l^2 + \dfrac{3Q^2}{2k_2^8 \exp(8lt)}\right\} \end{array}\right], \tag{59}$$

The pressure of the universe becomes

$$p = \frac{-1}{20(\lambda + 2\pi)(\lambda + 5\pi)} \left[\begin{array}{l} (\lambda + 10\pi)\left\{6l^2 - \dfrac{3Q^2}{4k_2^8 \exp(8lt)} - \dfrac{6\alpha^2}{k_2^8 \exp(2lt)}\right\} \\ + \dfrac{3}{2}(3\lambda + 10\pi)\left\{4l^2 + \dfrac{3Q^2}{2k_2^8 \exp(8lt)}\right\} \end{array}\right]. \tag{60}$$

VII. Concluding Remark

This paper is devoted to study the expansion of universe in $f(R,T)$ theory of gravity. We have obtained two five dimensional exact solutions of Bianchi type-V space time in $f(R,T)$ theory using the assumption of constant deceleration parameter and variation law of Hubble parameter. These solutions lead to two different models of the universe. The first solution represents to a singular model for $n \neq 0$ with power law expansion and second solution gives a non-singular model for $n = 0$ with exponential expansion of the universe.

i) five dimensional Singular model of the universe for $n \neq 0$

1) The metric coefficients *A,B,C* are zero at $t = -\frac{k_1}{nl}$. This shows that the model has point type singularity.

2) At $t = -\frac{k_1}{nl}$, volume scale factor V vanishes.

3) The mean generalized Hubble parameter H, expansion scalar θ, shear scalar σ^2 and mean anisotropy parameter $\overline{A}$ are all infinite at this point of singularity.

Above observational data suggest that universe starts its expansion with zero volume and it will continue to expand.

ii) five dimensional Non-singular model of the universe for $n = 0$

1) The metric coefficients A, B, C and volume of the universe increase exponentially with the cosmic time.
2) This five dimensional model of the universe is non-singular because of exponential behavior of the model and there is no singularity.
3) Mean generalized Hubble parameter H and expansion scalar θ are constant. Shear scalar σ^2 and anisotropy parameter $\overline{A}$ are finite for finite values of t.

This shows that the universe expansion take place with zero volume from infinite past.

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